lar Cones at Incidence," Rept. LR-385, NCR-7633, 1963, National Research Laboratories, Ottawa, Canada.

[3] Föppl, L., "Wirbelbewegung hinter einem Kreiszylinder," *Sitzungsberichte der Bayerischen Akademie der Wissenschaften zu Munchen*, Munich, Germany, 1913, pp. 1–17.

[4] Milne-Thomson, L. M., *Theoretical Hydrodynamics*, 4th ed., Macmillan Co., New York, 1960, pp. 367–369.

[5] Seath, D. D., "Equilibrium Vortex Positions in the Wake of Circular and Elliptic Bodies," Rept. AE 70-01, 1970, The University of Texas at Arlington Engineering Research, Arlington, Texas.

[6] Lin, C. C., "On the Motion of Vortices in Two Dimensions," Applied Mathematics Series No. 5, 1943, University of Toronto, Toronto, Canada.

[7] Seath, D. D., "Vortex Wake Investigation of An Inclined 15-Degree Circular Cone At Low Speeds," 1969, *The University of Texas at Arlington Engineering Research Report*, Arlington, Texas.

[8] Schlichting, H., *Boundary Layer Theory*, 4th ed., McGraw-Hill, New York, 1960, p. 252.

Estimation of the Maximum Temperature of a Swept Leading Edge for an Equilibrium Glide Entry

Davis H. Crawford*
NASA Langley Research Center, Hampton, Va.

INTEREST in the space shuttle with a large lateral range capability has prompted several designers to propose swept- or delta-winged configurations. A simple relation for estimating the maximum stagnation line temperature on the wing leading edge during an equilibrium glide entry is derived herein. This relation, amenable to hand calculation, is based on a modified form of the Detra, Kemp, and Riddell relationship[1] for the laminar heating (Btu/ft²-sec) to the stagnation point of a sphere of radius R:

$$q_{s,0} = 868(\rho/\rho_0 R)^{1/2}(10^{-8}u^2)^{1.575}\xi \tag{1}$$

where $\xi = [2C_p(T_\infty - T_w) + u^2]/[2C_p(T_\infty - 540) + u^2]$ and where ρ, ρ_0, C_p, and T_∞ are the local density, the sea-level density, the specific heat, and the temperature of the air, and where T_w and u are the vehicle wall temperature and the flight velocity. The atmospheric density variation, $\rho = \rho_k e^{-\beta a}$, is that suggested by Chapman in Ref. 2 where the reference density, $\rho_k = 0.087$ lb/ft³, together with a value of $\beta = 1/23{,}500$ provides satisfactory accuracy in the altitude range a of current interest. The velocity (fps) determined by equating the sum of the aerodynamic lift and centrifugal forces to the weight of the vehicle, is

$$u^2 = [(R_E + a)gB]/[(R_E + a)\rho/2 + B] \tag{2}$$

where R_E is the radius of the earth (20.9×10^6 ft), and $B = W/SC_L$, where W, S, and C_L are the weight, planform area, and lift coefficient of the vehicle.

The stagnation-line heating on a swept cylinder is related to the stagnation-point heating on a sphere by the factor $2^{1/2}$, and the function $\cos^{1.1}\Lambda_e$, Refs. 3–4, where Λ_e, the effective sweep angle, is $\sin^{-1}(\cos\alpha \sin\Lambda)$. Thus, the heat transfer to the swept-wing leading edge appears as follows:

$$q_s = q_{s,0}(\cos^{1.1}\Lambda_e)/2^{1/2} \tag{3}$$

At maximum heating the derivative of q_s with respect to a in Eq. (1) or Eq. (3) vanishes (ρ and u are functions of a, but the temperature variation is trivial). The corresponding values of altitude and velocity are

$$a = -(1/\beta)\ln[(2g/\rho_k u^2)B\eta]\text{ft} \tag{4}$$

where

$$\eta = [\beta(R_E + a) - (3.15 + 2\varphi)]/\{[\beta(R_E + a) - 1](3.15 + 2\varphi)\}$$

$$\varphi = [2C_p(T_w - 540)u^2]/[2C_p(T_\infty - T_w) + u^2][2C_p(T_\infty - 540) + u^2]$$

and

$$u^2 = g\zeta \tag{5}$$

where

$$\zeta = [\beta(R_E + a)^2](2.15 + 2\varphi)/[\beta(R_E + a) - 1](3.15 + 2\varphi)$$

When the maximum convected heat transfer from Eq. (3) is set equal to the radiated heat transfer, the stagnation-line temperature is given by

$$T_w^4 = (868/\sigma)[(2g/\rho_0 u^2)\eta]^{1/2}(10^{-8}u^2)^{1.575}\xi(B/R\epsilon^2)^{1/2} \times (\cos^{1.1}\Lambda_e)/2^{1/2} \tag{6}$$

where σ and ϵ are the Stefan-Boltzmann radiation constant and the emissivity, and the sea-level density is given[5] as $\rho_0 = 0.0765$ lb/ft³.

Iteration is necessary to evaluate u, a, and T_W. The convergence is rapid since the terms which call for iteration have a minor effect, and average values can be used with little error. For instance, the velocity u at which maximum heating occurs varies only in the narrow range from 21,650 fps to 21,680 as W/SC_L is varied from 40 to 320; similarly, a and T_W vary over a narrow span as a function of W/SC_L.

The foregoing discussion suggests the simplification of Eqs. (4–6) by evaluating constant terms, by selecting nominal values of terms which vary slightly, and by dropping negligible terms. With an assumed wall temperature of 3000°R and an assumed stream temperature of 400°R, the following simplified relations for peak heating may be found:

$$u = 21{,}650 \text{ fps} \tag{7}$$

$$a = 233{,}200 - 23{,}500 \ln(10^{-2}B)\text{ft} \tag{8}$$

$$T_w = 3202(10^{-2}B/R\epsilon^2)^{1/8} \cos^{0.275}\Lambda_e {}^\circ\text{R} \tag{9}$$

for a swept-cylinder leading edge, and

$$T_w = 3492(10^{-2}B/R\epsilon^2)^{1/8} {}^\circ\text{R} \tag{10}$$

for a sphere stagnation point. Thus, with $B = 100$ lb/ft², $R = 1$ ft, $\epsilon = 1.0$, and $\Lambda_e = 0$, the maximum equilibrium tem-

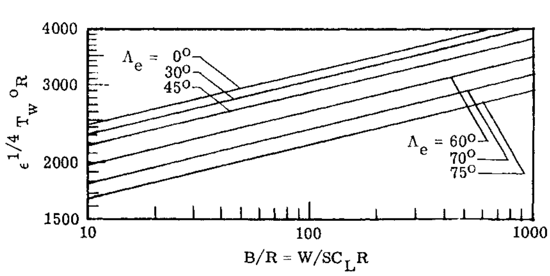

Fig. 1 Maximum stagnation-line temperatures for equilibrium-glide entry.

Received July 29, 1970; revision received September 4, 1970.

* Aerospace Engineer, Aerothermal Branch, Space Systems Research Division. Member AIAA.

perature of 3202°R will occur at an altitude of 233,200 ft. When the wing is swept 60°, the maximum leading-edge temperature is 2646°R and the maximum spherical-nose temperature is 3492°R. These temperatures differ from those calculated by the iterative procedure by less than 14°, which is an accuracy far greater than that inherent in the original calculated method of Ref. 1. Since the Re/ft range is from 4000 to 30,000 the assumption of laminar flow is well justified.

The preceding analysis assumes the vehicle to hold a constant altitude at each station of study. Along an inclined flight path with a resultant velocity U, with components u and v in the circumferential and radial directions, and with a resultant aerodynamic force inclined at an angle θ to the vertical, we have

$$T_w{}^4 = (868/\sigma)\{[2g(1+\delta)/\rho_0 U^2]\eta\lambda\}^{1/2}(10^{-8}U^2)^{1.575} \times \xi(B/R\epsilon^2)^{1/2}(\cos^{1.1}\Lambda_e)/2^{1/2} \quad (11)$$

where

$$\delta = v^2/g(R_E + a) \text{ and } \lambda = (L/D)/[1 + (L/D)^2 \cos\theta]^{1/2}$$

The equations showing the velocity and altitude where the maximum possible temperature occurs become

$$U^2/(1+\delta) = g\zeta \quad (12)$$

$$a = -(1/\beta)\ln\{[2g(1+\delta)/\rho_k U^2]B\eta\lambda\}\text{ft} \quad (13)$$

Note that these equations in which the higher-order terms have been ignored are similar in form to Eqs. (4–6). If $\theta \approx 1°$ ($v \approx 0.0175u$), the difference in calculated T_w introduced by neglecting these terms is less than 0.001%; if $\theta \approx 10°$ ($v \approx 0.176u$), the difference is less than 0.05%.

With small error (well within the uncertainty of the reference calculation method), Eqs. (11–13) may be simplified to the following:

$$U/(1+\delta)^{1/2} = 21{,}650 \text{ fps} \quad (14)$$

$$a = 233{,}200 - 23{,}500 \ln(10^{-2}B\lambda)\text{ft} \quad (15)$$

$$T_w = 3202(10^{-2}B/R\epsilon^2)^{1/8}(\cos^{0.275}\Lambda_e)(1+\delta)^{0.394}\lambda^{1/8}{}^\circ\text{R} \quad (16)$$

and for the hemisphere nose

$$T_w = 3492(10^{-2}B/R\epsilon^2)^{1/8}(1+\delta)^{0.394}\lambda^{1/8}{}^\circ\text{R} \quad (17)$$

Examination of Eqs. (14–17) shows that these are related to Eqs. (7–10) by the functions $(1+\delta)$ and $\lambda = (L/D)/[1 + (L/D)^2 \cos\theta]^{1/2}$. For entry at a small θ with high L/D, these terms are near 1.0 and have little effect. For a rapid estimate of maximum equilibrium temperature, the results of Eq. (9) are presented in Fig. 1. For steeper descent, the temperature will be changed by the factor $(1+\delta)^{0.394}\lambda^{1/8}$.

References

[1] Detra, R. W., Kemp, N. H., and Riddell, F. R., "Addendum to Heat Transfer for Satellite Vehicle Re-Entering the Atmosphere," *Jet Propulsion*, Vol. 27, Dec. 1957, pp. 1256–1257.

[2] Chapman, D. R., "An Approximate Analytical Method for Studying Entry Into Planetary Atmospheres," TR R-11, 1959, NASA.

[3] Wisniewski, R. J., "Methods of Predicting Laminar Heat Rates on Hypersonic Vehicles," TN D-201, 1959, NASA.

[4] Beckwith, I. E. and Gallagher, J. J., "Local Heat Transfer and Recovery Temperatures on a Yawed Cylinder at a Mach Number of 4.15 and High Reynolds Numbers," TR R-104, 1969, NASA.

[5] *U.S. Standard Atmosphere*, NASA, U.S. Air Force, and U.S. Weather Bureau, Washington, D. C., 1962.

Flexibility Coefficients for Structural Joint Assemblies

Saul M. Kaplan*

General Electric Company, Schenectady, N. Y.

Introduction

THE sharp, localized reduction in effective modulus of rigidity (EI) across structural joint assemblies significantly influences the natural frequencies and mode shapes of launch vehicles, missiles, and spacecraft. The modular construction generally featured in unmanned spacecraft, for example, will result in structural joint flexibilities that can cumulatively reduce the natural frequencies associated with principal structural resonances by 10–25%.

Structural joint bending flexibility data have been published by Alley and Ledbetter.[1] Their empirical results, as shown in Fig. 1, present joint rotation constants, or bending flexibilities, as functions of: 1) the diameter of the structural joint assembly; and 2) the nature of joint construction. The four categories of joints—excellent, good, moderate, and loose—identified by Alley and Ledbetter, have their flexibility values presented as rather wide scatter bands. These structural joint flexibility data, while originally conceived as being applicable to multistage launch vehicles and missiles, have been successfully applied to a wide and diverse spectrum of other structures, including spacecraft, deployable spacecraft antennae, solar panels, and other complex spacecraft subsystems. Some high levels of correlation between structural dynamic analytical predictions and subsequent

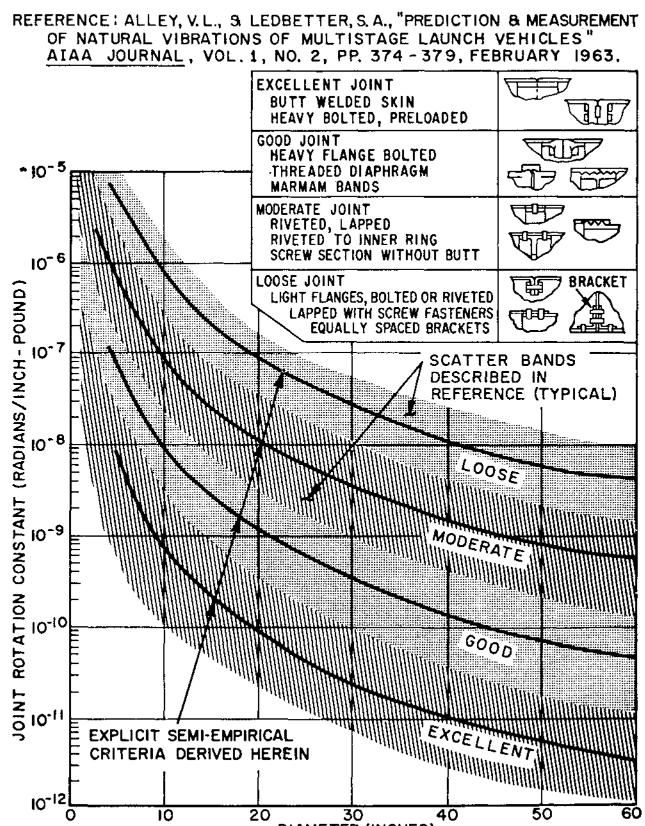

Fig. 1 Alley and Ledbetter's joint rotation constants.[1]

Received October 8, 1970. This work was performed while the author was at the Company's Space Division, Valley Forge, Pennsylvania.

* Project Engineer. Gas Turbine Department.